

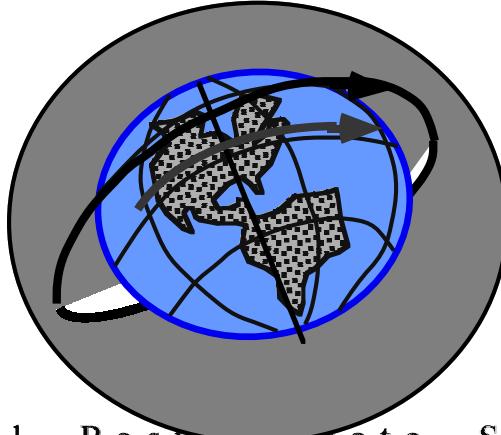
SS3011

SS3011 Space Technology and Applications

“Orbitology” (cont'd)

Homework on Orbital Velocity

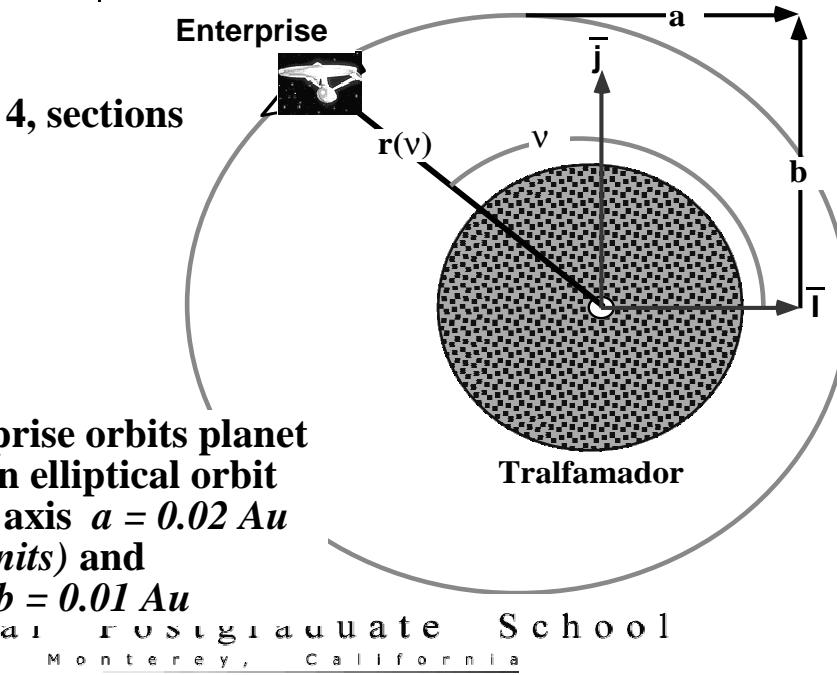
SS3011 Space Technology
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Homework elliptical orbits: 2



- Sellers, Chapter 4, sections 4.1, 4.2, 4.3

- Starship Enterprise orbits planet *Tralfamador* in an elliptical orbit with semi-major axis $a = 0.02 \text{ Au}$ (*astronomical units) and semi-minor axis $b = 0.01 \text{ Au}$

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Homework elliptical orbits: 2

- Given that Tralfamador has a mass 100 times greater than the Earth
- i) Compute the period of the orbit (check your units!)

Hint 1

$$\mu_{\text{earth}} = G M \approx 6.672 \times 10^{-11} \frac{\text{Nt}\cdot\text{m}^2}{\text{kg}^2} \times 5.974 \times 10^{24} \text{kg} =$$

$$3.98565 \times 10^{14} \frac{\text{Nt}\cdot\text{m}^2}{\text{kg}} = 3.986 \times 10^{14} \frac{\text{m}^3}{\text{sec}^2} = 1.4076 \times 10^{16} \frac{\text{ft}^3}{\text{sec}^2}$$

***AU \simeq 150,000,000 kilometers**

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Homework elliptical orbits: 2

- Calculate the Magnitude of the velocity at
 - i) perifamador
 - ii) apfamador

Hint 2

$$\frac{r_{\max} + r_{\min}}{2} = a$$

$$|\bar{V}_p|^2 = \mu \left[\frac{2}{r_p} - \frac{1}{a} \right]$$

$$\frac{r_{\max} - r_{\min}}{r_{\max} + r_{\min}} = e$$

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Homework

elliptical orbits: 2

- Calculate the ΔV required to achieve "Escape velocity" at
 - perifamador
 - apfamador

Hint 3: i.e. find the change in velocity that puts the satellite on an elliptical trajectory with

$$a = \infty$$

$$V_{\text{escape}}^2 = \lim_{a \rightarrow \infty} \left[\frac{2 \mu}{r} - \frac{\mu}{a} \right] = \frac{2 \mu}{r}$$

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First Compute the Period of the Orbit

$$a = 0.02 \text{ AU} = 0.02 \times [150 \times 10^6] \text{ km} = [3.0 \times 10^6] \text{ km}$$

$$\mu_{\text{tralfamador}} = G M \approx 6.672 \times 10^{-11} \frac{\text{Nt}\cdot\text{m}^2}{\text{kg}^2} \times 5.974 \times 10^{26} \text{ kg} =$$

$$3.98565 \times 10^{16} \frac{\text{Nt}\cdot\text{m}^2}{\text{kg}} = 3.986 \times 10^7 \frac{\text{km}^3}{\text{sec}^2}$$

SS3011 Compute the Period of the Orbit

(cont'd)

Kepler Third Law

$$T = \frac{2\pi a^{3/2}}{\sqrt{\mu}}$$

$$a = [3.0 \times 10^6] \text{ km}$$

$$\mu_{\text{tralfamador}} = 3.986 \times 10^7 \frac{\text{km}^3}{\text{sec}^2}$$

$$T = \frac{2\pi a^{3/2}}{\mu} = \frac{6.283185 \times [3.0 \times 10^6 \text{ km}]^{3/2}}{\sqrt{3.986 \times 10^7 \frac{\text{km}^3}{\text{sec}^2}}} =$$

$$6.283185 \times \frac{5,196.2 \times 10^6 \text{ km}^{3/2}}{6313.5 \frac{\text{km}^{3/2}}{\text{sec}}} = 5171.2 \times 10^6 \text{ sec} = 59.852 \text{ days}$$

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SS3011 From Earlier Homework Problem Homework:Solution

- Minimum and maximum distances

perifamador, -- Minimum distance :

↓

$$v = 0, \pm 2n\pi \Rightarrow r_{\min} = a \frac{1 - e^2}{[1 + e \cos(\pm 2n\pi)]} = a \frac{1 - e^2}{[1 + e]} = a \frac{[1 + e][1 - e]}{[1 + e]} =$$

$$a[1 - e] = (0.02 \text{ au})[1 - 0.866] = 0.00268 \text{ au}$$

apfamador, -- Maximum distance :

↓

$$v = (\pm 2n+1)\pi \Rightarrow r_{\max} = a \frac{1 - e^2}{[1 + e \cos((\pm 2n+1)\pi)]} = a \frac{1 - e^2}{[1 - e]} = a \frac{[1 + e][1 - e]}{[1 - e]} =$$

$$a[1 + e] = (0.02 \text{ au})[1 + 0.866] = 0.03732 \text{ au}$$

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Velocity at Perifamador

$$r_{\min} = 0.00268 \text{ au} \times [150 \times 10^6] \text{ km} = 402,000 \text{ km}$$

$$|\bar{V}_{r_{\min}}| = \sqrt{\mu \left[\frac{2}{r_{\min}} - \frac{1}{a} \right]} =$$

$$\sqrt{3.986 \times 10^7 \frac{\text{km}^3}{\text{sec}^2} \left[\frac{2}{402,000 \text{ km}} - \frac{1}{[3.0 \times 10^6 \text{ km}]} \right]} =$$

13.602 km/sec

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Escape Velocity at Perifamador

$$r_{\min} = 0.00268 \text{ au} \times [150 \times 10^6] \text{ km} = 402,000 \text{ km}$$

$$|\bar{V}_{r_{\min}}|_{\text{escape}} = \sqrt{\mu \left[\frac{2}{r_{\min}} \right]} =$$

$$\sqrt{3.986 \times 10^7 \frac{\text{km}^3}{\text{sec}^2} \left[\frac{2}{402,000 \text{ km}} \right]} =$$

14.08 km/sec

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ΔV required to escape from orbit at Perifamador

$$|\Delta V_{r_{\min}}|_{\text{escape}} = |V_{r_{\min}}|_{\text{escape}} - |V_{r_{\min}}| =$$

$$[14.080 - 13.602] \frac{\text{km}}{\text{sec}} = 0.478 \text{ km/sec}$$

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Velocity at Apfamador

$$r_{\max} = 0.03732 \text{ au} \times [150 \times 10^6] \text{ km} = 5,598,000 \text{ km}$$

$$|V_{r_{\max}}| = \sqrt{\mu \left[\frac{2}{r_{\max}} - \frac{1}{a} \right]} =$$

$$\sqrt{3.986 \times 10^7 \frac{\text{km}^3}{\text{sec}^2} \left[\frac{2}{5,598,000 \text{ km}} - \frac{1}{[3.0 \times 10^6 \text{ km}]} \right]} =$$

$$0.9769 \text{ km/sec}$$

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SS3011 Escape Velocity at Apfamador

$$r_{\max} = 0.03732 \text{ au} \times [150 \times 10^6] \text{ km} = 5,598,000 \text{ km}$$

$$|\bar{V}_{r_{\max}}|_{\text{escape}} = \sqrt{\mu \left[\frac{2}{r_{\max}} \right]} =$$

$$\sqrt{3.986 \times 10^7 \frac{\text{km}^3}{\text{sec}^2} \left[\frac{2}{5,598,000 \text{ km}} \right]} =$$

3.7737 km/sec
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ΔV required to escape from orbit at Perifamador

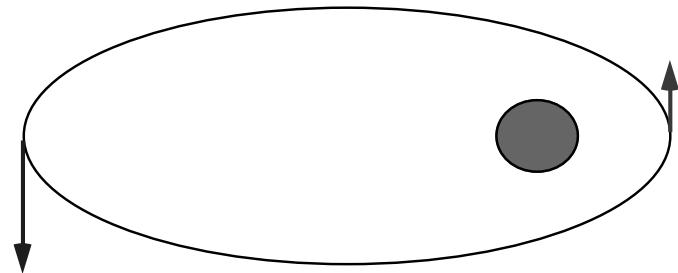
$$|\Delta V_{r_{\max}}|_{\text{escape}} = |\bar{V}_{r_{\max}}|_{\text{escape}} - |\bar{V}_{r_{\max}}| =$$

$$[3.7737 - 0.9769] \frac{\text{km}}{\text{sec}} = 2.7968 \text{ km/sec}$$

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Comparison of escape " ΔV 's"

$$|\Delta V_{r_{\min}}|_{\text{escape}} = 0.9769 \text{ km/sec}$$



$$|\Delta V_{r_{\max}}|_{\text{escape}} = 2.7968 \text{ km/sec}$$

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